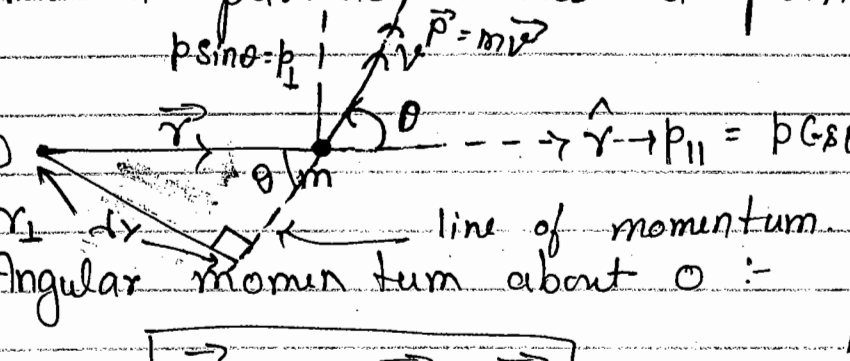
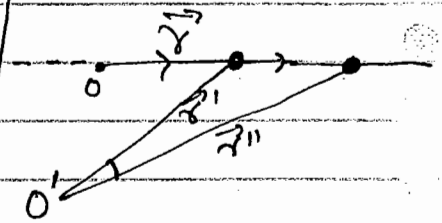
L Rigid body dynamics is important topic for JEE]

* Angular Momentum And Torque :-

{ for a particle, about a point } :-





Angular momentum about O :-

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \Rightarrow \text{Vector form}$$

$\vec{r}$ = position of particle from point O.

Use right hand thumb rule to find the direction of $\vec{L}$.

Magnitude form :-

$$\boxed{|\vec{L}| = rp\sin\theta = rp_\perp}$$

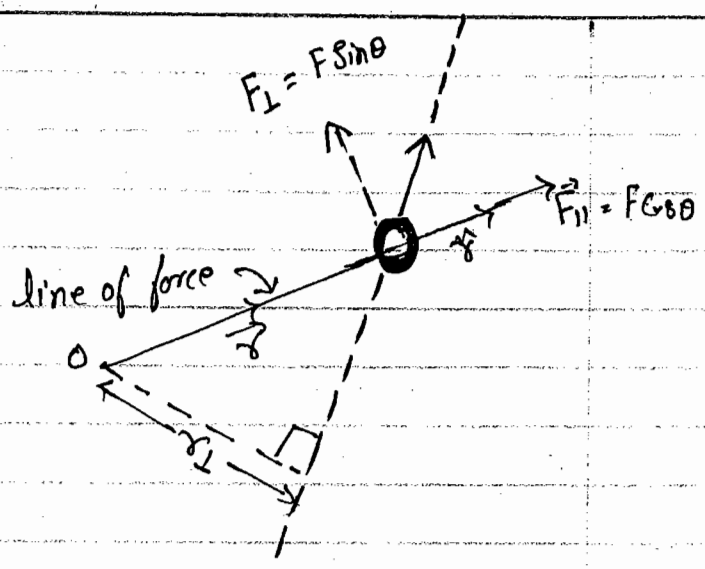
$$(\because p_\perp = p\sin\theta)$$

$$\therefore \boxed{|\vec{L}| = r_\perp p}$$

$$(\because r_\perp = r\sin\theta)$$

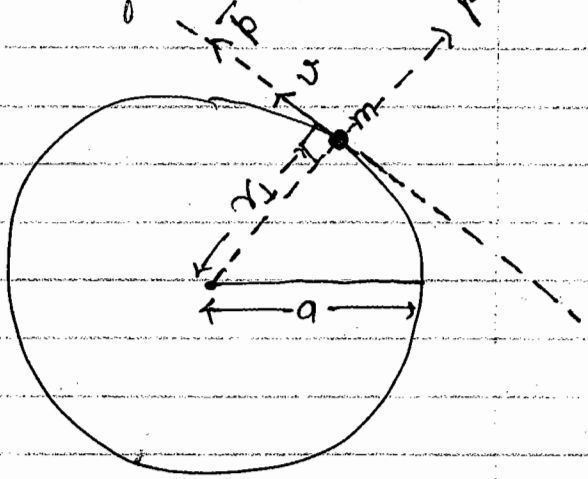
$$\therefore \boxed{L = rp\sin\theta = rp_\perp = pr_\perp}$$

* Torque :-



Qu A particle is moving in a circle of radius 'a' with constant speed v calculate its angular momentum about its centre of the circle.

Sol^n



$$L = p\, r_\perp$$

$$\boxed{L = mva}$$

$(\because p = mv)$ $r_\perp = a$

for Direction -

$$\vec{L} = \vec{r} \times \vec{p}$$ {finger curl from r to p}

So direction is up the plane of paper. Up the plane of paper.

Qu A particle of mass m is moving with speed v_0 along a straight line $y = (\alpha x + \beta)$ $\alpha > 0$ $\beta > 0$ in x-y plane. Calculate angular momentum about origin.

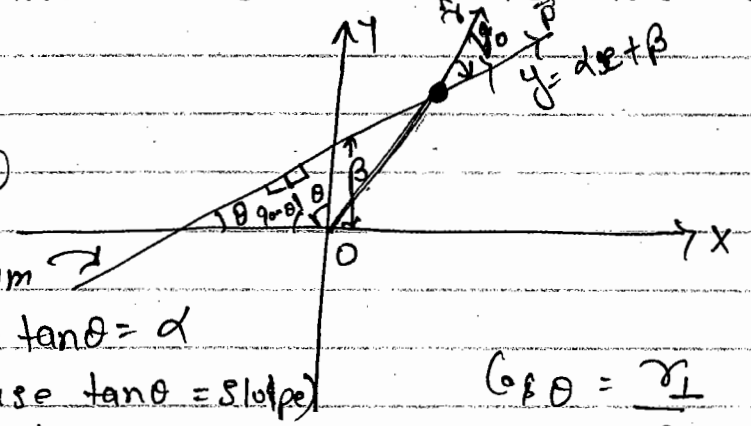
Sol^{n}

$$y = \alpha x + \beta$$

α → slope, β → intersept

Slope is +ve that means angle from +ve x direction is less than 90°.

Intersept is +ve that means it cuts the +ve y-axis.



$$L = p r_\perp \quad \text{—①}$$

→ line of momentum (Particle moving about this line)

$\tan\theta = \alpha$ (because $\tan\theta$ = slope which is α.)

$$\cos\theta = \frac{r_\perp}{\beta}$$

$$\therefore \tan\theta = \frac{\alpha}{1} = \frac{\text{Perpendicular}}{\text{Base}} \quad \therefore r_\perp = \beta\cos\theta$$

$$\cos\theta = \frac{r_\perp}{\beta} = \frac{\text{Base}}{\text{Hypoten.}}$$

$$\text{So} \quad r_\perp = \beta \cdot \frac{\text{Base}}{\text{Hypoten.}} = \beta \cdot \frac{1}{\sqrt{\alpha^2+1}}$$

$$\text{So} \quad \boxed{r_\perp = \frac{\beta}{\sqrt{\alpha^2+1}}}$$

$$\text{So} \quad \boxed{L = m v_0 \frac{\beta}{\sqrt{\alpha^2+1}}} \quad \{\{\text{from ①}\}\}$$

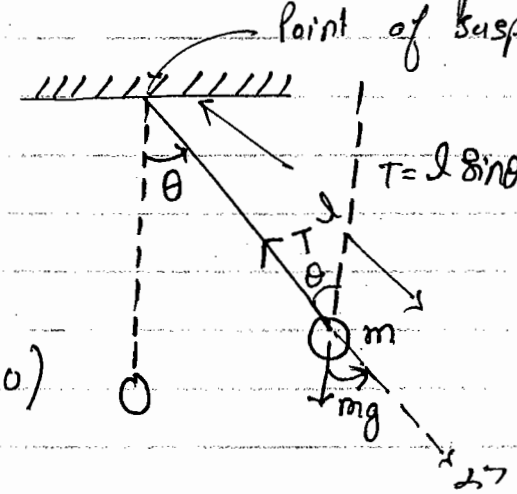
direction of angular momentum is into the page or $(-\hat{z})$ axis.

$r \times p$

Qus A simple pendulum consisting of a small bob of mass m and light string of length l is deflected by an angle θ from lowest position. Calculate torque about point of suspention in this position.

Sol^n

$$\tau = F r_\perp$$



① Due to tension :-

$$\tau = T \cdot 0 = 0 \quad (\because r_\perp = 0)$$

$$\boxed{\tau = 0}$$

If a force passes through a point so that force will not produce any torque about point.

② Due to mg :-

$$\tau = mg \cdot l \sin\theta$$

$$\boxed{\tau = mgl \sin\theta} \quad (r_\perp = l \sin\theta)$$

Direction of τ is into the plane of paper.

Qus What is torque on mass $3m$ due to forces of other masses about point O ?